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DROUGHT INDICATORS AND TRIGGERS: A STOCHASTIC APPROACH TO EVALUATION[1]

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ABSTRACT: Drought management depends on indicators to detect drought conditions, and triggers to activate drought responses. But determining those indicators and triggers presents challenges. Indicators often lack spatial and temporal transferability, comparability among scales, and relevance to critical drought impacts. Triggers often lack statistical integrity, consistency among drought categories, and correspondence with desired management goals. This article presents an approach for developing and evaluating drought indicators and triggers, using a probabilistic framework that offers comparability, consistency, and applicability. From that, a multistate Markov model investigates the stochastic behavior of indicators and triggers, including transitioning, duration, and frequency within drought categories. This model is applied to the analysis of drought in the Apalachicola-Chattahoochee-Flint River Basin in the southeastern United States, using indicators of the Standardized Precipitation Index (for 3, 6, 9, and 12 months), the Palmer Drought Severity Index, and the Palmer Hydrologic Drought Index. The analysis revealed differences among the performance of indicators and their trigger thresholds, which can influence drought responses. Results contribute to improved understanding of drought phenomena, statistical methods for indicators and triggers, and insights for drought management.
(KEY TERMS: drought; indicators; triggers; stochastic; Markov model; Standardized Precipitation Index; Palmer Index; Apalachicola-Chattahoochee-Flint River Basin.)

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INTRODUCTION

Drought is common yet not commonly understood. Drought conditions can be difficult to define, and drought indicators and triggers can lack scientific justification. Yet sound indicators and triggers are important to detect the onset of drought conditions, to monitor and measure drought events, and to reduce drought impacts.

While the complexity of drought has been well noted (e.g., Dracup *et al.,* 1980), approaches to develop drought indicators and triggers are still needed. Water managers grapple with questions concerning which indicators to use, and which trigger values to set for each indicator. In many cases, multiple indicators are used, but multiple indicators on multiple scales can confound the complexity of single indicators. Even standardized indicators may use scales and categorical thresholds that are spatially and temporally inconsistent (Karl *et al.,* 1987; Guttman *et al.,* 1992), incomparable with other indicators, or difficult to interpret and apply.

This paper provides an approach for expressing indicators within a probabilistic framework, and for evaluating their stochastic properties using a multistate homogenous Markov model. The usefulness of this approach becomes apparent when comparing, combining, and choosing among drought indicators, and determining trigger values. It offers an equitable basis for evaluation, ease of interpretation, and direct application to water management decisions.

This model then investigates the performance of six indicators in a study of drought in the Apalachicola-Chattahoochee-Flint (ACF) River Basin, central to the "Tri-State Water Wars" – the federal lawsuit concerning water allocation among the States of Georgia, Alabama, and Florida. The determination of drought indicators and triggers for the ACF basin has become a focus of the Water Wars, as this determines the timing and type of management actions, such as granting relief from the required flow targets.

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More generally, by using the model to evaluate and quantify critical properties of drought indicators, decisions can be based on justifiable and statistical criteria rather than arbitrary or inconsistent criteria.

DIMENSIONS OF INDICATORS AND TRIGGERS

A drought indicator, briefly defined, is a variable to identify and assess drought conditions. Common indicators are based on meteorologic and hydrologic variables such as precipitation, streamflow, soil moisture, reservoir storage, and ground water levels. A drought trigger is a threshold value of the drought indicator that distinguishes a drought category, and determines when drought response actions should begin or end. Drought categories typically represent levels of severity, such as "mild, moderate, severe, or extreme drought." For example, for an indicator of "streamflow," a drought trigger could be "streamflow below the 5th percentile for one month," which could then invoke the category of "extreme drought, " and a corresponding set of management responses.

Because drought depends on numerous factors, such as water supplies and demands, hydrologic and political boundaries, and antecedent conditions, indicators should be sensitive to context. More than 150 drought definitions have been published (Wilhite and Glantz, 1985), and each one could conceivably generate a suite of relevant indicators.

The challenge then becomes how to select indicators to represent and quantify drought conditions, and how to establish triggers to achieve the desired management goals. Important questions arise, such as: How much will the indicator oscillate between drought categories, or remain in the same drought category? How frequently will the trigger be invoked? How long will it stay in effect, once invoked? How do these triggers relate to measures of drought severity?

To compound this challenge, indicators often lack a consistent statistical basis for the triggers that determine drought categories. For example, the category of "extreme drought," as defined by the Palmer Drought Severity Index (PDSI), includes values of less than or equal to -4.00 (Palmer, 1965; Karl, 1986). Yet this category has varying probability of occurrences, depending on location and time, ranging from less than 1 percent in January in the Pacific Northwest to more than 10 percent in July in the Midwest (Karl *et al.,* 1987; Guttman *et al.,* 1992; Lohani et al., 1998).

In addition, even statistically consistent indicator scales can be difficult to directly apply and combine with other indicators. For example, the Standardized Precipitation Index (SPI) (McKee *et al.,* 1993), whose indicator thresholds are based on the statistical Z-score, has varying probability differentials for equal index differentials. The probability differential between an SPI of -1.0 and -1.5 is 9.1 percent, and between an SPI of -1.5 and -2.0 is 4.4 percent, for instance, even though both represent an index differential of 0.5, and it is these index differentials that define SPI drought categories. Moreover, drought categories are not necessarily comparable among indicators. The category of "extreme drought" occurs less than 4 percent of the time for the PDSI, considering all months and climate divisions (Karl, 1986), but less than 2.3 percent of the time for the SPI (McKee *et al.,* 1993).

The approach in this paper transforms indicators to an equivalent categorization system based on percentiles. Once indicator data are transformed (as described later in this paper), and categories of drought severity are defined according to threshold probabilities, the Markov model is applied to describe and interpret the indicators in terms of their transitioning, persistence, duration, and frequency within categories. The motivation for this approach is to provide a flexible and equitable basis for using and evaluating one or more drought indicators, determining trigger values, and linking triggers to drought categories that are comparable among indicators.

STOCHASTIC CHARACTERISTICS OF INDICATORS

To analyze drought indicators and triggers, a multistate Markov chain is developed to represent the time correlation of random variables (drought indicators) that can take on the value of two or more states (drought indicator categories). Consider a stochastic process $\{J_n: n = 1, 2, \ldots\}$ with s categories $\{1, \ldots, s\}$ where J_n represents the value of one of s possible drought indicator categories during the nth time period. For each time period, the value of J_{n+1} can either remain in the same category as in the previous time period, J_n, or it can change to one of the other categories. Assume that this stochastic process characterizes the Markovian property, whereby the probability of the future category J_{n+1} depends only on the current category, J_n, but not on previous categories $J_{n-1},\ldots,J_1$. This characteristic of first-order discrete time Markov chains can be expressed more formally as

$$\Pr\{J_{n+1} \mid Jn, J_{n-1},\ldots,J_1\} = \Pr\{J_{n+1} \mid J_n\} \qquad (1)$$

Thus, the conditional probabilities for the category of J_{n+1} depend only on the category of J_n. While the conditional probabilities of the future category depend only on the current category, the value of the future category can nonetheless depend on prior categories; that is, the categorical value of J_{n+1} can depend not only on the value of J_n, but also on $J_{n-1}, J_{n-2},...,J_1$.

The Markovian property implies conditional independence of values separated by more than one time period, but there can still be statistical dependence among values in a Markov series. For example, Pr $\{J_{n+1} = 1 \mid J_n = 2, J_{n-1} = 1, J_{n-2} = 2,...,J_1 = 3\}$ = Pr $\{J_{n+1} = 1 \mid J_n = 2\}$, but it is possible that Pr $\{J_{n+1} = 1 \mid J_{n-2} = 2\} \neq$ Pr $\{J_{n+1} = 1 \mid J_{n-2} = 3\}$. If the conditional probabilities are independent of the time period under consideration (n), then the Markov chain is said to be stationary or homogeneous in time. That is, for all categories i and j: Pr $\{J_{n+1} = j \mid J_n = i\}$ = Pr $\{J_{n+1+t} = j \mid J_{n+t} = i\}$ for t = {-(n), -(n-1), -(n-2),...,-1, 0, 1, 2,...}. The Markov chain is said to be nonstationary (or nonhomogeneous) if the conditional probabilities depend on the time period (n) under consideration.

Many drought indicators exhibit trends or cycles, and are not inherently stationary; however, nonstationary data can often be processed to make a reasonable assumption of stationarity. One approach is to stratify the data into subsets that approximate stationarity, and conduct separate analyses with these subsets. For instance, given a long term record of monthly indicator values, the data for each month across all years can be analyzed separately, rather than analyzing all months from all years collectively. Another approach is to transform the data by removing influences of location and spread from the dataset, such as by subtracting periodic means and dividing by periodic standard deviations. For instance, normalized indicator values can be transformed to a statistical Z-score, which is a standardized anomaly with constant mean and standard deviation. Both of these approaches were employed for the study in this article.

The performance of indicators in the Markov process can be described by "transition probabilities," which are the conditional probabilities of being in a certain category, J_{n+1}, for the future time period n+1, given a certain category, J_n, for the present time period n. Let p_{ij} represent the transition probability that J_n will be in category j at time n+1, given that J_n was in category i at time n, expressed as

$$p_{ij} = \Pr\{J_{n+1} = j \mid J_n = i\} \tag{2}$$

Estimates of the transition probabilities, $\hat{p}_{ij}$, can be calculated from the conditional relative frequencies of the transition counts, m_{ij}

$$\hat{p}_{ij} = \frac{m_{ij}}{\sum_j m_{ij}} \qquad i, j = 1,...,s \tag{3}$$

where m_{ij} = the frequency that J_n is in category i at time n, and category j at time n+1. The numerator represents the number of transitions from category i to category j, and the denominator is the sum of the number of transitions from category i to any other category. These parameter estimates consider the edge effect, meaning that the final point in the data series is not counted in the denominator because there is no data value that follows to be counted in the numerator.

Let $\underline{P} = (p_{ij})$ denote the matrix of transition probabilities for the multistate first order Markov chain. For an s-state Markov chain, the matrix will contain s^2 entries, where the sum of each row's entries will be equal to 1, or $(\sum_j p_{ij}) = 1$ for each value of i. A sample calculation of the transition probability matrix, as part of the modeling process described in the following section, is presented in the Appendix.

Drought Indicator Transitioning, Persistence, Duration, and Frequency

For the model developed in this paper, define the persistence probability, ξ_k, as the probability of remaining in a drought category k (k=1,...,s) from time n to time n+1

$$\xi_k = p_{ij} \qquad \text{where } k = i = j \tag{4}$$

where the values ξ_k are represented by the diagonal of the transition probability matrix.

Define υ_k as the random duration for which a drought indicator remains in a category k during t consecutive time periods. The probability of uninterrupted duration is

$$P(\upsilon_k = t) = \xi_k^{\,t-1}(1 - \xi_k) \tag{5}$$

for t-1 events of unchanging drought categories, followed by a change in drought categories. This is based on the geometric distribution, where (ξ_k) is the probability of remaining in the same drought category, and $(1-\xi_k)$ is the probability of changing drought categories. From this, the average duration becomes

$$\upsilon_k = \frac{1}{(1-\xi_k)} \tag{6}$$

which represents the average length of time (number of consecutive time periods) that a drought indicator will remain in drought category k.

Define Φ_k as the random frequency for which a drought indicator will fall within a certain category, k (k=1,...,s), and $\Phi = [\Phi_1, \Phi_2,...,\Phi_s]$ as the frequency vector determined by the relationship

$$(I-A^T)\,\Phi = 0 \tag{7}$$

where I is the identity matrix and A^T is the transpose of the transition probability matrix. This variable, Φ_k, represents the number of discrete time periods relative to total time periods, expressed as a percentage, that a drought indicator will be triggered in a certain category k.

Drought Indicator Categories and Thresholds Probabilities

The categories of drought (states of the stochastic process) can be defined according to threshold probabilities, τ_k, k (k=1,...,s), which represent cumulative probabilities, $F(x_k)$, of a particular drought indicator variable, such that

$$\tau_k = F(x_k) = \Pr\{X \le x_k\} \tag{8}$$

where X is a random value of the drought indicator, and x_k is the value of the drought indicator corresponding to the threshold probability for category k. The upper bound of a category is established by τ_k, and the lower bound by τ_{k+1}. This set of threshold probabilities is used to define trigger values for the categories of an indicator.

The statistical characteristics of various drought indicators can then be examined by using threshold probabilities to define drought indicator categories along a scale of cumulative probability. For example, for a six-state categorization (from the example in the Appendix)

$$\{\tau_1,...,\tau_6\} = \{1.0, 0.5, 0.35, 0.20, 0.10, 0.05\} \tag{9}$$

$$\{(J_n=1;\ 0.50 < p(x) \le 1.00);\ (J_n=2;\ 0.35 < p(x) \le 0.50);\ (J_n=3;\ 0.20 < p(x) \le 0.35);\ (J_n=4;\ 0.10 < p(x) \le 0.20);\ (J_n=5;\ 0.05 < p(x) \le 0.10);\ (J_n=6;\ 0.00 \le p(x) \le 0.05)\}.$$

In this example, drought severity increases with increasing values of k, such that $J_n = 1$ represents wet and near normal/wet conditions, $J_n = 2$ represents near normal/dry conditions, and $J_n = 3, 4, 5, 6$ represents mild, moderate, severe, and extreme drought, respectively. Note that this nomenclature is intended to illustrate rather than define. The less severe categories are generally associated with drought mitigation and response, rather than drought conditions *per se,* and the designation of drought is often reserved for the most severe categories.

This model based on cumulative probability offers a consistent basis for categorizing drought indicators and for comparing multiple indicators and triggers. Indicator data can be transformed to a scale based on percentiles, with each datum corresponding to a particular percentile and drought category as defined by threshold probabilities. The model can be readily adapted to any number of categories of drought, with any desired threshold probabilities, to accommodate any number of indicators. The case study in the next section illustrates the development, application, and implications of this model, using six categories and six indicators.

CASE STUDY: DROUGHT IN THE ACF RIVER BASIN

The ACF River Basin (Figure 1) is formed by the Apalachicola, Chattahoochee, and Flint Rivers in the southeastern United States (U.S.). The basin originates in the north Georgia mountains with the Chattahoochee River and in the south metropolitan Atlanta area with the Flint River. These rivers flow into Lake Seminole near the Georgia-Florida border, then into Florida as the Apalachicola River. The basin is approximately 385 miles (619 km) long and 50 miles (80 km) wide. Most of the ACF basin lies in Georgia (74 percent), with the remainder in Alabama (15 percent) and Florida (11 percent). The ACF basin has a semi-humid climate, with mean annual precipitation of approximately 60 inches (152.4 cm) at the north and south ends, and 45 inches (114.3 cm) at the east central area. Water demands in the basin include municipalities, industry, agriculture, hydropower, navigation, fish and wildlife habitat, flood control, water quality, and recreation (USACE, 1998).

Droughts in the southeastern U.S. have accentuated public concern about water availability and management in the basin. In May 1990, the U.S. Army Corps of Engineers (USACE) proposed the reallocation of reservoir storage for water supply in north Georgia, and the State of Georgia submitted plans for a water supply reservoir approximately five miles upstream from the Alabama-Georgia state line. The State of Alabama filed a lawsuit against the USACE, challenging the proposed water reallocations, and the State of Florida joined the fray. In efforts to resolve the conflict, the States of Alabama, Georgia, and Florida agreed to the Comprehensive Study of the Apalachicola-Chattahoochee-Flint (ACF) and Alabama-Coosa-Tallapoosa (ACT) River Basins, which led to the ACF River Basin Compact, whose purpose is to develop an allocation formula, with a 50-year planning horizon, for equitably apportioning the surface waters of the ACF basin among the three

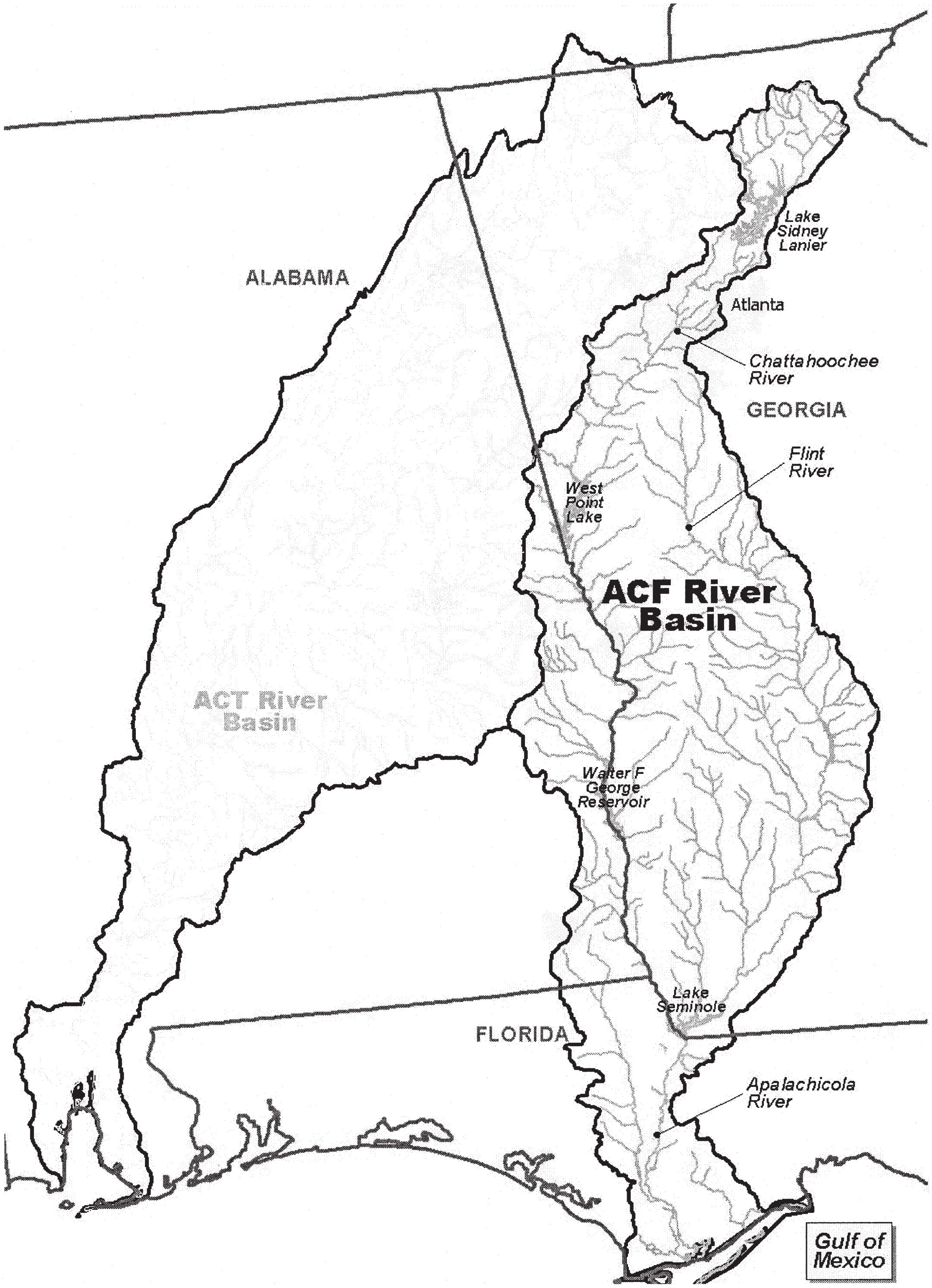

Figure 1. The Apalachicola-Chattahoochee-Flint (ACF) and Alabama-Coosa-Tallapoosa (ACT) River Basins (modified from USACE, 1998).

states.

The development of a drought plan for the ACF basin has become an integral and required part of the ACF agreement. The drought plan will involve procedures for identifying the onset and progression of drought stages using appropriate indicators, a tiered process of notices and mitigating actions, and procedures for identifying the recession and termination of drought stages (State of Florida, 2002; State of Georgia, 2002). The identification of drought conditions is also a part of the ACF allocation formula, in that drought triggers can determine when relief can be granted from the minimum flow and reservoir operation requirements (State of Florida, 2002; State of Georgia, 2002).

As part of the ACF study, representatives from the states considered several drought indicators, including the Palmer Hydrologic Drought Index (PHDI), and the 12-month Standardized Precipitation Index (SPI-12). The states decided that indicator values for the ACF basin would be calculated using a weighted aggregation of climate division values, based on areal extent of each climate division's relative contribution to the upstream (Alabama-Georgia) portion of the basin. Thus, the weighted ACF indicator value = $\sum [v_i \times w_i]$, where v_i = value of indicator for climate division i, and w_i = weight based on area of climate division relative to upstream basin area. For Georgia Climate Divisions 1, 2, 3, 4, 5, 7, and 8, these weights are 0.6, 10.7, 1.2, 34.1, 0.1, 32.3, and 4.6 percent, respectively. For Alabama Climate Divisions 5, 6, and 7, these weights are 4.1, 0.7, and 11.6 percent, respectively. The states also decided upon a 63-year study period (January 1939 to December 2001) for the analysis.

The study of drought indicators reported in this paper, however, undertook a broader investigation of indicators to compare and evaluate their performance. This study included two Palmer indices (PDSI and PHDI), and four SPI indicators based on 3, 6, 9, and 12-month anomalies (SPI-3, 6, 9, 12). For each of these six indicators in this investigation, 107 years of monthly data representing the long term record (1895 to 2001) were obtained and transformed to cumulative distribution functions for extracting percentiles and determining categorical thresholds (as detailed in the following sections).

A six-state Markov model was then applied to each of these indicators, using categories of drought as previously defined by the threshold probabilities, τ_k (k=1,...,6) = {1.00, 0.50, 0.35, 0.20, 0.10, 0.05}. These categories were selected for consistency with the drought plan currently under development for the State of Georgia. Yet any number of categories and corresponding percentile ranges could be similarly employed. Also, while the ACF study is ongoing, this model can be similarly adapted, providing an approach by which indicators can be analyzed, triggers selected, and drought criteria established. Accordingly, using this model, questions regarding the persistence, transitions, duration, and frequency of the drought indicators were investigated.

Palmer Drought Severity Index and Palmer Hydrologic Drought Index

The PDSI, based on the Palmer Drought Model (Palmer, 1965), is derived from principles of a moisture balance, using historic records of precipitation, temperature, and the local available water content of the soil. The PHDI uses a modification of the PDSI to assess moisture anomalies that affect streamflow, ground water, and water storage (Karl, 1986).

The PDSI is generally defined for a spell of dry weather by

$$\mathrm{PDSI}_i = 0.897\mathrm{PDSI}_{i-1} + (Z_i/3) \tag{10}$$

where i is the month of interest,
and Z is the moisture anomaly index which is given by

$$Z_i = (P_i - \hat{P}_i)K_i \tag{11}$$

where P_i is the observed precipitation for month i, $\hat{P}_i$ is the "climatologically appropriate precipitation for existing conditions" (CAFEC), and K_i is a weighting factor obtained by

$$K_i = \left(\frac{17.67}{\sum_{i=1}^{12} \bar{D}_i K_i'}\right) K_i' \tag{12}$$

where $\bar{D}_i$ is the average of the absolute values of $(P_i - \hat{P}_i)$ for month i during all years of record and K_i' is given by

$$K_i' = 1.5\log\left(\frac{\overline{PE}_i + \bar{R}_i + \overline{RO}_i}{\bar{P}_i + \bar{L}_i}\right) + 2.8\bar{D}_i^{-1} + 0.5 \tag{13}$$

where PE_i is potential evapotranspiration, R_i is soil water recharge, RO_i is runoff, P_i is precipitation, and L_i is water loss from the soil, for month i. The overbar denotes monthly averages for the period of record. The expression inside the parentheses can be viewed as the ratio of moisture demand to moisture supply for the month and region.

The determination of when a drought has ended is given by the computation of the "percentage probability," Pe_i, such that

$$Pe_i = \frac{\sum_{j=0}^{j=j^*} U_{i-j}}{Z_e + \sum_{j=1}^{j=j^*} U_{i-j}} \text{ X 100 percent} \tag{14}$$

where

$$U_i = Z_i + 0.15 \tag{15}$$

in the case of a drought, leading to

$$Z_e = -2.691(PDSI_{i-1}) - 1.5 \tag{16}$$

which is the Z-value in a single month that will end a drought, that is, bring the PDSI value to -0.5, based on Equation (10).

The primary difference between the PDSI and the PHDI is their beginning and ending times of a dry spell, based on Pe – the ratio of moisture received to moisture required to terminate a drought, where Pe is greater than or equal to zero and less than or equal to one. With the PDSI, the drought is considered to have ended when Pe is greater than zero. With the PHDI, however, the drought does not end until Pe is equal to one.

The drought categories of the PDSI/PHDI (Table 1) are based on Palmer's model (1965), with cumulative frequencies for all months and all climate divisions in the U.S. based on Karl (1986).

TABLE 1. Drought Categories for PDSI/PHDI.

PDSI/PHDI Values	Drought Category	Cumulative Frequency (approximate) (percent)
0.00 to -1.49	Near Normal	28 to 50
-1.50 to -2.99	Mild to Moderate Drought	11 to 27
-3.00 to -3.99	Severe Drought	5 to 10
-4.00 or less	Extreme Drought	< = 4

The cumulative frequencies associated with the index values for each drought category, however, vary according to region and time period under consideration (Karl *et al.,* 1987; Guttman *et al.,* 1992; Soulé, 1992; Nkemdirim and Weber, 1999). For instance, the probability of occurrence of the category of "extreme drought" (-4.00 or less) is greater than 10 percent (rather than less than 4 percent) in many regions of the country (such as the Pacific Northwest), and that probability also varies by month within a region (Guttman *et al.,* 1992).

The variability of the PDSI was also investigated by Lohani *et al.* (1998) and Lohani and Loganathan (1997), using a nonhomogenous Markov model. Here, categories were defined by PDSI thresholds, and transition probabilities varied and depended on the month and the climate division. Results confirmed differences in PDSI probabilities of occurrence, both temporally and spatially. For example, the PDSI category of "extreme drought" occurred in Virginia CD1, January, 4.17 percent, July, 2.08 percent; and in Virginia CD6, January, 3.12 percent, July, 1.04 percent.

Thus, a challenge in using the Palmer indices is that the categorical threshold values (such as -1.50, -3.00, etc.) are not necessarily consistent, in terms of probability of occurrence, either spatially or temporally. The variability among categories also hinders comparison of the Palmer indices with other indicators, as illustrated earlier with the SPI (see Alley, 1984; Karl *et al.,* 1987; Guttman, 1998; Hayes *et al.,* 1999).

The approach in this article converts the PDSI and PHDI values to percentiles, rather than using the raw index values and thresholds. The percentiles are determined through empirically derived statistics from a stratification of the long term record for each month and each climate division, from which stationary transition probabilities can be derived for the Markov model. Thus, in this homogeneous Markov approach, the transitional probabilities are independent of the month and the location. Using percentiles instead of raw indicator values also enables comparison of multiple drought indicators and their stochastic characteristics.

To transform the PDSI and PHDI into percentiles, historical monthly PDSI and PHDI values for 107 years (1895 to 2001), obtained from the National Climatic Data Center (NCDC), were used to develop an empirical cumulative distribution function (ECDF) for each month, using estimates of p(x) constructed from the following ranking procedure, $p(x_i) = \frac{i}{(n+1)}$, where x is the value of the drought indicator, i is the rank of the order statistic, x_i, where i = 1,...,n, and n is the number of data values (see, Harter, 1994; Piechota and Dracup, 1996). Thus, the smallest data value in the sample is $x_{(1)}$, and the largest data value in the sample is $x_{(n)}$. Once the ECDFs were generated for each month and each climate division, percentile values were determined for each PDSI and PHDI value for each month and each climate division. From that, drought category values (i.e., 1,…,6) were associated with each percentile value for each month, and the Markov model was applied.

Standardized Precipitation Index

The SPI is a standardized anomaly, equivalent to the statistical Z-score, representing the precipitation deficit over a specific time scale, such as 3, 6, or 12 months, relative to climatology (McKee *et al.,* 1993). The calculation of the SPI begins with the transformation of a long term record of precipitation data (typically 30 years or more, but in this model, 107 years) to a standard normal distribution. One common procedure is to fit a gamma distribution to the data, although the Pearson III has also been recommended (Guttman, 1999), and then to transform the data to an equivalent SPI value based on the standard normal distribution. To begin, the gamma distribution is defined by the probability density function

$$g(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)} \quad (17)$$

with x, α, $\beta > 0$, where α is a shape parameter; β is a scale parameter; x, for this context, is precipitation amount; and $\Gamma(\alpha)$ is the gamma function, defined by

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1}e^{-y}dy \quad (18)$$

The two parameters of the distribution, α and β, are estimated for each station, for each time scale, and for each month of the year. The maximum likelihood approximations, using Thom (1958), are given by

$$\hat{\alpha} = \frac{1}{4A}\left(1+\sqrt{1+\frac{4A}{3}}\right) \quad (19)$$

and

$$\hat{\beta} = \frac{\bar{x}}{\hat{\alpha}} \quad (20)$$

where A is the sample statistic

$$A = \ln(\bar{x}) - \frac{\frac{1}{n}\sum_{i=1}^{n}\ln(x_i)}{n} \quad (21)$$

which is the difference between the logs of the arithmetic and geometric means.

The cumulative probability is given by

$$G(x) = \int_0^x g(x)dx = \frac{1}{\hat{\beta}^{\hat{\alpha}}\Gamma(\hat{\alpha})}\int_0^x x^{\hat{\alpha}-1}e^{-x/\hat{\beta}}dx \quad (22)$$

which can be expressed as

$$G(x) = \frac{1}{\Gamma(\hat{\alpha})}\int_0^x t^{\hat{\alpha}-1}e^{-t}dt \quad (23)$$

where

$$t = x/\hat{\beta},$$

Because the gamma function is undefined for x equal to zero, and precipitation may be equal to zero, the cumulative probability becomes

$$H(x) = q + (1 - q)\,G(x) \quad (24)$$

where q is the probability of zero precipitation, which can be estimated by m divided by n if m is the number of zeros (Thom, 1958).

Climatological data for monthly total precipitation and SPI values for a period of 107 years (1895 to 2001) were obtained from the NCDC and the Western Regional Climate Center (WRCC), and transformed to cumulative probabilities for the Markov model through this process. To calculate the SPI, the values of the variate (precipitation) from the fitted distribution (in this case, gamma) are transformed to values of the variate on a prescribed distribution (in this case, standard normal), so that the probability of being less than a given value of the variate is the same as the probability of being less than the corresponding value of the transformed variate (following Panofsky and Brier, 1958). From this, the statistical Z-score (SPI value) can be assigned to each of the percentile values. Similarly, given SPI values, the associated percentiles can be directly determined, as these correspond to the statistical Z-score percentiles.

The categories of the SPI, according to McKee *et al.* (1993), are shown in Table 2.

TABLE 2. Drought Categories for SPI.

SPI Values	Drought Category	Cumulative Frequency (percent)
0 to -0.99	Near Normal	16 to 50
-1.00 to -1.49	Mild to Moderate Drought	6.8 to 15.9
-1.50 to -1.99	Severe Drought	2.3 to 6.7
-2.00 or less	Extreme Drought	< 2.3

TABLE 3. Drought Indicator Transition Probabilities, p_{ij}, Based on the Six-State Markov Model for ACF Basin Indicators for the Study Period (1939 to 2001).

	State "j"					
State "i"	1	2	3	4	5	6
			PHDI			
1	0.892	0.092	0.014	0.002	0.000	0.000
2	0.315	0.444	0.234	0.008	0.000	0.000
3	0.075	0.280	0.398	0.247	0.000	0.000
4	0.000	0.029	0.235	0.500	0.221	0.015
5	0.000	0.100	0.250	0.400	0.100	0.150
6	0.000	0.000	0.000	0.083	0.083	0.833
			PDSI			
1	0.882	0.106	0.012	0.000	0.000	0.000
2	0.315	0.414	0.252	0.018	0.000	0.000
3	0.088	0.154	0.451	0.308	0.000	0.000
4	0.054	0.068	0.203	0.446	0.203	0.027
5	0.211	0.000	0.053	0.368	0.158	0.211
6	0.000	0.000	0.038	0.192	0.000	0.769
			SPI-3			
1	0.777	0.120	0.061	0.031	0.007	0.005
2	0.486	0.143	0.219	0.105	0.038	0.010
3	0.313	0.208	0.198	0.219	0.042	0.021
4	0.132	0.224	0.289	0.171	0.092	0.092
5	0.097	0.065	0.194	0.323	0.194	0.129
6	0.048	0.000	0.000	0.333	0.381	0.238
			SPI-6			
1	0.837	0.122	0.041	0.000	0.000	0.000
2	0.395	0.306	0.218	0.073	0.008	0.000
3	0.157	0.255	0.353	0.176	0.039	0.020
4	0.048	0.113	0.258	0.355	0.129	0.097
5	0.000	0.080	0.160	0.320	0.280	0.160
6	0.000	0.000	0.083	0.208	0.208	0.500
			SPI-9			
1	0.894	0.079	0.023	0.005	0.000	0.000
2	0.327	0.376	0.257	0.040	0.000	0.000
3	0.122	0.224	0.429	0.184	0.041	0.000
4	0.030	0.091	0.227	0.424	0.152	0.076
5	0.000	0.000	0.143	0.429	0.321	0.107
6	0.000	0.000	0.053	0.105	0.263	0.579
			SPI-12			
1	0.892	0.090	0.018	0.000	0.000	0.000
2	0.376	0.410	0.205	0.009	0.000	0.000
3	0.038	0.333	0.423	0.192	0.013	0.000
4	0.016	0.049	0.213	0.459	0.213	0.049
5	0.000	0.000	0.028	0.361	0.444	0.167
6	0.000	0.000	0.000	0.158	0.316	0.526

Although the SPI can represent different temporal and spatial scales on a statistically comparable basis, meaning that an SPI value is the same in terms of cumulative probability across time periods and locations, the SPI values themselves can be difficult to apply directly. For instance, a change of -0.5 in the SPI value can represent a probability change of 9.1 percent (upper and lower bounds for moderate drought) or a change of 4.4 percent (upper and lower bounds for severe drought). The nomenclature and percentiles associated with the SPI value can also be inconsistent with other indices. For instance, in the PDSI/PHDI, an index value of -1.49 corresponds to a percentile of 28 percent, and a lower bound of "near normal," whereas with the SPI, an index value of -1.49 corresponds to a percentile of 6.8 percent, and a lower bound of "moderate drought." This provides additional rationale for the use of percentiles for developing, comparing, and evaluating triggers.

RESULTS: EVALUATION OF ACF BASIN INDICATORS

The Markov model was used to analyze each of six drought indicators for the ACF basin, using percentiles relative to each month based on the long term record (January 1895 to December 2001), for the 63-year study period (January 1939 to December 2001), and for the six categories defined earlier in Equation (9). Results are presented in Tables 3 through 6. This section provides an interpretation of the model results, and discussion of the decision making implications.

Drought Indicator Transitioning, Persistence, Duration, and Frequency

The matrices of transition probabilities (Table 3) address the question: What is the probability that a given indicator, currently in drought category "i," will be in drought category "j" for the next time period? The analysis of transition probabilities can be used for short term and long term planning, and the probabilistic characterization of the progression and recession of drought. For example, assume the current category is Category 5. For the SPI-3, p_{5j} = {0.097, 0.065, 0.194, 0.323, 0.194, and 0.129}, whereas for SPI-12, p_{5j} = {0.000, 0.000, 0.028, 0.361, 0.444, and 0.167}. For the SPI-3, the most probable category for the next time period would be moving to Category 4 (32.3 percent), with a lesser probability (19.4 percent) of remaining in Category 5 or transitioning to Category 3, and even lesser probabilities (9.7, 6.5, and 12.9 percent) of transitioning to Categories 1, 2, and 6, respectively. Yet for the SPI-12, the most probable category for the next time period would be remaining in Category 5 (44.4 percent), a lesser probability (36.1 percent) of transitioning to Category 4, even lesser probabilities (2.8 and 16.7 percent) of transitioning to Categories 3 and 6, respectively, and a zero probability (0.0 percent) for Categories 1 and 2. Thus, the SPI-3 exhibits greater oscillation among drought categories [e.g., 9.7 percent probability of transitioning from a severe drought (Category 5) to wet/near normal conditions (Category 1) within a month]; whereas the SPI-12 exhibits less oscillation and more stability around its current category [e.g., 0.0 percent probability of transitioning from a severe drought (Category 5) to wet/near normal conditions (Category 1)].

Next, consider the persistence probabilities (Table 4), which address the question: What is the probability that the drought category for the next time period will be the same as the current drought category? Maximum values of ξ_k (k=1,…,6) occurred for ξ_1 SPI-9; ξ_2 PHDI; ξ_3 PDSI; ξ_4 PHDI; ξ_5 SPI-12; and ξ_6 PHDI, meaning that these indicators have the highest persistence for each drought category during the study period. (The persistent probabilities in Table 4 represent the diagonal values of the transition probability matrices in Table 3.) The PHDI's relatively high persistence can be explained, in part, because the indicator tends to respond slowly to short term changes. For the SPI-12, the indicator is based on a 12-month moving average, and thus will be less sensitive to monthly changes, and similarly the nine-month basis for the SPI-9. The indicator with the minimum value of ξ_k for nearly all categories is the SPI-3, consistent with its shorter averaging period (three months) and greater oscillation relative to the other indicators.

Now consider duration. This addresses the question: Once a certain drought category is triggered, what is the average length of time that it will remain triggered? For duration (Table 5), maximum values of υ_k (k=1,…,6) were consistent with maximum values of ξ_k, given their mathematical relationship. Values of υ_k for the SPIs generally followed the magnitude of their time scale (3, 6, 9, or 12 months), with the SPI-3 having the shortest duration, and the SPI-9 or SPI-12 having the longest duration. For k = 1, 5, maximum values of υ_k occurred for the SPI-9 and SPI-12 respectively, and for k = 2, 3, 4, 6 for the PHDI and PDSI. For instance, for the category of extreme drought (k = 6), υ_6 = 1.313 months for the SPI-3, yet υ_6 = 6.0 months for the PHDI, meaning it remains triggered, on average, more than four times as long. Values of υ_k also depend on the probabilistic range of the category, based on Equation (9) (e.g., Category 1 is 50 percent, Category 6 is 5 percent), which relates to frequency.

TABLE 4. Drought Indicator Persistence Probabilities, ξ_k, Based on the Six-State Markov Model for ACF Basin Indicators for the Study Period (1939 to 2001).

	1	2	3	4	5	6
PHDI	0.892	0.444	0.398	0.500	0.100	0.833
PDSI	0.882	0.414	0.451	0.446	0.158	0.769
SPI-3	0.777	0.143	0.198	0.171	0.194	0.238
SPI-6	0.837	0.306	0.353	0.355	0.280	0.500
SPI-9	0.894	0.376	0.429	0.424	0.321	0.579
SPI-12	0.892	0.410	0.423	0.459	0.444	0.526

TABLE 5. Drought Indicator Durations, υ_k (months), Based on the Six-State Markov Model for ACF Basin Indicators for the Study Period (1939 to 2001).

	1	2	3	4	5	6
PHDI	9.261	1.797	1.661	2.000	1.111	6.000
PDSI	8.510	1.708	1.820	1.805	1.188	4.333
SPI-3	4.484	1.167	1.247	1.206	1.240	1.313
SPI-6	6.147	1.442	1.545	1.550	1.389	2.000
SPI-9	9.426	1.603	1.750	1.737	1.474	2.375
SPI-12	9.250	1.696	1.733	1.848	1.800	2.111

Frequency addresses the question: What is the probability that an indicator will trigger a certain drought category during a certain time period? For these six indicators (Table 6), and this 63-year study period (1939 to 2001), values of Φ_1 for each indicator were greater than categorical values (i.e., the percentile ranges for each category, based on the long term record, 1895 to 2001), and values of Φ_3, Φ_4, Φ_5, and Φ_6 were less than or equal to the categorical values for all indicators. For Φ_2, the PHDI, SPI-6, and SPI-12 were greater than the categorical value, whereas the PDSI, SPI-3 and SPI-9 were less than the categorical value. This means that, overall, dry conditions were less frequent during the 63-year study period, relative to the long term record, even though discretized periods exhibited more frequent dry conditions. For instance, half-decadal analyses found extreme drought conditions were more frequent during the period 1951 to 1955, with Φ_6 = 19.7, 23.0, 6.6, 14.8, 13.1, and 16.4 percent for the PHDI, PDSI, SPI-3, SPI-6, SPI-9, and SPI-12, respectively, whereas the categorical value is 5 percent. Thus, frequency analyses can also help to delineate and compare periods of drought, and categorize drought severity.

TABLE 6. Drought Indicator Frequencies, Φ_k, Based on the Six-State Markov Model for ACF Basin Indicators for the Study Period (1939 to 2001).

	1 (%)	2 (%)	3 (%)	4 (%)	5 (%)	6 (%)
PHDI	56.3	16.4	12.3	9.1	2.6	3.2
PDSI	57.4	14.7	12.0	9.9	2.5	3.4
SPI-3	56.4	13.9	12.7	10.0	4.2	2.8
SPI-6	55.3	16.4	13.5	8.2	3.3	3.3
SPI-9	58.6	13.4	13.0	8.8	3.7	2.5
SPI-12	58.7	15.5	10.4	8.1	4.8	2.5
Categorical	50.0	15.0	15.0	10.0	5.0	5.0

Implications for Drought Management

There are several decision making implications of these results. First, concerning transition probabilities and persistence, while these analyses can determine whether an indicator is more persistent or more oscillatory than other indicators, determining the degree of persistence that is desired in an indicator depends on the decision and the decision maker. Some water managers prefer an indicator to remain in a certain category of drought, once triggered, for at least a certain period of time; otherwise, it could cause confusion and lack of credibility if that category and associated management responses were frequently invoked and revoked. Other water managers prefer an indicator that would be more sensitive to short-term changes, and easily invoked and revoked, to make sure that drought conditions were addressed with timely responses.

The transition probabilities, combined with information on duration, can also help to determining whether an indicator would be an early warning or a false alarm of drought progressing or receding. That is, as drought progresses, is the indicator value an early warning of long term drought, or is it an artifact of a short term deficit? As drought recedes, is the indicator value a sign of long term recovery, or of a short term surplus? While definitions of drought vary widely, as do criteria for early warnings and false alarms, the analyses can nonetheless help to characterize the sequencing and probability of categories of drought severity.

Consider, for instance, indicators that invoke Category 4 (moderate drought). For the SPI-3, p_{4j} = {0.132, 0.224, 0.289, 0.171, 0.092, and 0.092}, and for the PHDI, p_{4j} = {0.000, 0.029, 0.235, 0.500, 0.221, and 0.015}. This indicates a 64.5 percent probability that the SPI-3 will transition to a less severe category (Category 1, 2, or 3) in the next time period, and a 26.4 percent probability that the PHDI will move to a less severe category. Even if the SPI-3 or PHDI were to transition from Category 4 to a less severe category, such as Category 1, 2, or 3, each indicator could nonetheless transition back to Category 4 or a more severe category in the subsequent time period, with a probability of 47.8 percent (for the SPI) and 25.7 percent (for the PHDI) respectively, based on values of p_{ij}, (i = 1, 2, 3; j = 4, 5, 6). Thus, a decision tree of possible outcomes, such as drought category triggering or cumulative precipitation deficits (see, Lohani and Loganathan, 1997), and their associated probabilities can be generated for any number of future time periods by using the transition probabilities.

Duration concerns how long a drought trigger is likely to remain in a certain category, once it is invoked, as the time period associated with persistence. Some water managers prefer indicators with a longer duration as to incur less risk of invoking a certain drought category, only to revoke that drought category soon after. Other water managers prefer indicators with a shorter duration to pick up anomalous periods of dryness that may be precursors to longer term drought. Duration is also relevant for triggers that are defined for multiple time periods. As an example, for the currently proposed State of Georgia Drought Plan, to invoke a certain category of drought, an indicator needs to be in a certain (or more severe) category of drought for two or more consecutive months, and to revoke a certain category of drought, all indicators need to be in a certain (or less severe) category of drought for four or more consecutive months. These triggers are intended to alert and guide decision makers, however, rather than automatically invoke and revoke statewide drought responses (Steinemann, 2003).

Frequency analyses can be used both retrospectively and prospectively to establish drought triggers, compare drought indicators, and characterize drought severity. For example, given a desired frequency of triggering of drought responses, an historical analysis of indicators can reveal the threshold values that would correspond to that frequency, which then can provide a basis for trigger values in a drought management plan. Frequency analysis can also determine if drought triggers and categorical definitions are on parity; that is, if multiple indicators are used, to determine if the threshold values for each category would trigger at the same or desired frequency. In addition, frequencies can delineate periods of drought conditions, and characterize the severity of those conditions, by comparing categorical triggering. Frequency information can be considered along with transitioning and duration to assess trigger behavior. For instance, the long term SPI-3 and SPI-12 would have the same theoretical frequency of triggering a category, yet the patterns of triggering are typically quite different: the SPI-3 is more intermittent, whereas the SPI-12 is more persistent. In this study, the model analyzed all six indicators according to the same categorical scale, based on percentiles, so that they were comparable in terms of frequencies or probability of occurrence. But the same model could also be used to evaluate indicators on different categorical scales to see which ones would have been triggered more frequently, as the example below will demonstrate.

ACF Basin Triggers Evaluation

To extend this evaluation, the Markov model was used to characterize proposed drought triggers for the ACF study negotiations. In the proposal dated January 11, 2002, conditions for drought relief were based on single trigger values: (a) The ACF basin weighted SPI-12 less than -1.40, or (b) The ACF basin weighted PHDI less than -2.29. Note that, in this case, the categorical thresholds were based on index values rather than percentiles. A two-state Markov model was applied, where Category 0 meant the trigger was not invoked, and Category 1 meant the trigger was invoked. Indicators were based on the same 63-year study period (1939 to 2001) for the ACF basin. Results are presented in Table 7.

This analysis revealed that, using the proposed index values as triggers, the PHDI trigger would be invoked more frequently, and would remain invoked longer on average, than the SPI-12. The PHDI was triggered 11.9 percent of the time, with an average duration of 3.9 months, whereas the SPI-12 was triggered 5.3 percent of the time, with an average duration of 3.1 months. Although durations were comparable, the frequencies differed appreciably. For the study period, the PHDI would have triggered relief for 90 months, whereas the SPI-12 would have triggered relief for 40 months – less than half of the PHDI.

TABLE 7. Markov Model Results for Proposed Drought Triggers for the ACF Basin Compact Study (PHDI ≤ -2.29; SPI-12 ≤ -1.40) for the Study Period (1939 to 2001).

State "i"	State "j" 0	State "j" 1
PHDI (Category 0 ≥ -2.29, Category 1 < -2.29)		
Transition Probabilities		
0	0.967	0.033
1	0.256	0.744
Duration		
	30.2	3.9
Frequency / Total (percent)		
0	88.1	
1	11.9	
SPI-12 (Category 0 ≥ -1.40, Category 1 < -1.40)		
Transition Probabilities		
0	0.982	0.018
1	0.325	0.675
Duration		
	55.0	3.1
Frequency / Total (percent)		
0	94.7	
1	5.3	

Differences concerning the probability of triggering by specific months were also investigated. Whereas the SPI-12 probabilities based on the long-term record (associated with the SPI value of -1.40, with a cumulative probability of 8.08 percent) are consistent for each month, the PHDI probabilities based on the long term record (associated with the index value of -2.29) vary by month. Table 8 shows the percentiles associated with PHDI values (-4.0, -3.0, -1.5, and 0.0) for the ACF basin, based on the long term record. For example, for an index value of -4.0 or less ("extreme drought") for January, the cumulative probability is 1.2 percent, whereas for July, the cumulative probability is 3.2 percent. Table 8 also shows that, for each of the months, the cumulative probabilities vary from those reported in Karl (1986).

TABLE 8. Empirical Cumulative Probabilities for the PHDI, According to Month and Index Thresholds, for the ACF Basin, Based on the Historic Record (1895 to 2001).

	PHDI			
	-4.0 (percent)	-3.0 (percent)	-1.5 (percent)	0.0 (percent)
January	1.2	7.8	30.4	51.4
February	1.8	4.8	24.9	50.9
March	2.3	3.7	31.1	51.3
April	2.0	4.2	32.2	54.6
May	1.7	6.6	30.4	48.0
June	1.7	7.4	29.3	49.4
July	3.2	8.1	29.5	51.1
August	2.6	7.0	29.9	56.2
September	2.7	8.9	29.0	51.3
October	2.8	9.0	29.2	50.8
November	3.1	6.6	26.5	54.3
December	1.4	8.0	28.7	57.3

To analyze this variability, and to place the triggers on a statistically comparable basis, the long term record of indicator data for the PHDI and SPI-12 were transformed into percentiles, and compared by month. Table 9 shows, in the first column of numerical data, the PHDI cumulative probability associated with the index value of -2.29 for each month and, in the second column of numerical data, the PHDI value for each month that would correspond to the same cumulative probability (8.08 percent) of triggering as the SPI-12 value of -1.40. This analysis revealed an inconsistency with the proposed trigger values for the ACF study. The PHDI trigger of -2.29 was set higher (triggered more frequently) than the SPI value of -1.40, and that triggering frequency also varied by month. The triggers are currently being reevaluated, and a more complete evaluation of indicators is being performed for the ACF study.

This leads to a more general question about the selection and combination of indicators for representing drought conditions. The Palmer indices may not adequately represent droughts affecting managed water systems; one reason is that water supply storage is not directly considered in the index. The SPI is based on only precipitation, and droughts are often influenced by other factors (such as demand); although the SPI can capture such factors indirectly, such as reduced demand for outdoor water use because of increased precipitation. Whereas the SPI-3 is indicative of shorter term precipitation anomalies, and can be an early warning of potential long-term drought, it is also more oscillatory and can also cause more frequent invoking and revoking of drought responses. The SPI-12 reflects longer term dryness, as do the PHDI and PDSI, and may respond more slowly to incipient drought conditions, yet it is also more persistent and stable. The SPI-6 and SPI-9 provide intermediate indicators between the SPI-3 and the SPI-12 and Palmer indices. The point is that a single indicator of drought may often be insufficient. If multiple indicators are used, they should be transformed to a consistent scale, such as percentiles, and evaluated according to metrics, such as those investigated by the Markov model, that will clarify, inform, and justify their use in decision making.

TABLE 9. Comparison of (a) Monthly Trigger Probabilities for the PHDI Value of -2.29; and (b) Monthly PHDI Values for the Trigger Probability (8.08 percent) Associated With the SPI Value of -1.40, Both for the ACF Basin, Based on the Historic Record (1895 to 2001).

	(a) -2.29 (PHDI value)	(b) 8.08 (percent)
January	14.4	-2.92
February	11.9	-2.62
March	16.2	-2.59
April	15.4	-2.67
May	19.9	-2.82
June	20.4	-2.88
July	17.2	-3.00
August	19.1	-2.83
September	19.7	-3.05
October	15.6	-3.05
November	16.1	-2.85
December	14.9	-2.99

SUMMARY

Drought has multiple dimensions, and this paper presents an approach for comparing, combining, and choosing among multiple drought indicators and triggers. It offers a framework based on percentiles, which provides not only spatial and temporal comparability, but also intuitive and direct application to water management decisions. From this, a multistate Markov model was developed to evaluate drought indicators and their performance according to characteristics of transition probabilities, persistence, duration, and frequency within drought severity categories. The model is adaptable to any number of drought category definitions, and any range of percentiles. While the model can provide quantitative results, the criteria for what is desirable in indicators and triggers, such as degree of persistence, depends on the decision-making context. Important criteria independent of context, however, are that indicators and triggers should be understandable to the public and decision makers, statistically sound and defensible, and evaluated for their performance under progressing, continuing, and receding drought conditions.

APPENDIX

EXAMPLE OF CALCULATION OF MARKOV TRANSITION PROBABILITY MATRIX

Step 1: Obtain raw indicator data. In this example, the indicator is the SPI-3 for the period of January 1990 to December 2000 for the ACF Basin (Table A1).

Step 2: Determine percentiles associated with the raw values of the indicators (Table A2), as detailed in the article, through a stratification or transformation.

Step 3: Determine the drought category J_n = s (n = 1, 2,…; s = 1,2,...,6) associated with the percentile value (Table A2), according to the categorical thresholds shown in Table A3.

TABLE A1. SPI-3 Raw Data for January 1990 to December 2000.

SPI-3 Raw Values	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
January	1.198	0.633	0.282	1.788	0.166	-0.429	0.755	0.334	1.458	-0.465	-0.465
February	1.244	0.491	0.771	0.183	-0.161	0.085	0.333	0.838	1.400	-0.848	-1.255
March	1.031	1.033	0.798	0.637	0.354	0.087	1.011	0.218	1.142	-0.747	-0.811
April	0.244	0.276	-0.068	0.008	0.178	-0.210	0.534	0.209	1.304	-1.657	-1.232
May	0.067	1.462	-0.941	-0.001	-0.035	-1.179	0.480	-0.153	0.691	-1.238	-1.216
June	-1.021	1.464	-0.713	-1.317	0.373	-0.676	-0.647	0.799	0.030	-0.155	-1.612
July	-1.066	1.456	-0.033	-1.627	2.450	-0.961	-1.067	0.175	-0.778	0.104	-2.046
August	-1.785	0.667	0.879	-1.817	3.299	-0.272	-0.692	-0.470	-0.815	-0.269	-1.395
September	-1.612	-0.161	0.729	-1.385	2.788	-0.749	0.189	-0.470	0.950	-1.512	-0.263
October	-0.983	-1.048	0.619	0.037	1.665	1.245	0.467	0.647	0.770	-0.809	-0.107
November	-0.797	-1.264	1.665	0.526	1.066	1.236	0.211	1.631	0.574	-0.308	0.736
December	-0.476	-0.972	1.628	0.390	0.568	1.310	-0.171	1.802	-1.380	-0.378	0.109

TABLE A2. Percentile Values for the SPI-3 for January 1990 to December 2000.

SPI-3 Percentiles	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
January	0.88	0.74	0.61	0.96	0.57	0.33	0.77	0.63	0.93	0.32	0.32
February	0.89	0.69	0.78	0.57	0.44	0.53	0.63	0.80	0.92	0.20	0.10
March	0.85	0.85	0.79	0.74	0.64	0.53	0.84	0.59	0.87	0.23	0.21
April	0.60	0.61	0.47	0.50	0.57	0.42	0.70	0.58	0.90	0.05	0.11
May	0.53	0.93	0.17	0.50	0.49	0.12	0.68	0.44	0.76	0.11	0.11
June	0.15	0.93	0.24	0.09	0.65	0.25	0.26	0.79	0.51	0.44	0.05
July	0.14	0.93	0.49	0.05	0.99	0.17	0.14	0.57	0.22	0.54	0.02
August	0.04	0.75	0.81	0.03	1.00	0.39	0.24	0.32	0.21	0.39	0.08
September	0.05	0.44	0.77	0.08	1.00	0.23	0.58	0.32	0.83	0.07	0.40
October	0.16	0.15	0.73	0.51	0.95	0.89	0.68	0.74	0.78	0.21	0.46
November	0.21	0.10	0.95	0.70	0.86	0.89	0.58	0.95	0.72	0.38	0.77
December	0.32	0.17	0.95	0.65	0.72	0.90	0.43	0.96	0.08	0.35	0.54

TABLE A3. Drought Category Thresholds.

Category	Percentile Range
1	0.50 to 1.00
2	0.35 to 0.50
3	0.20 to 0.35
4	0.10 to 0.20
5	0.05 to 0.10
6	0.00 to 0.05

$\{(J_n = 1; 0.50 < p(x) \leq 1.00); (J_n = 2; 0.35 < p(x) \leq 0.50); (J_n = 3; 0.20 < p(x) \leq 0.35); (J_n = 4; 0.10 < p(x) \leq 0.20); (J_n = 5; 0.05 < p(x) \leq 0.10); (J_n = 6; 0.00 \leq p(x) \leq 0.05)\}$

Step 4: Calculate transition probability matrix by calculating the number of times that a drought level $J_n = i$ is followed by a drought level $J_{n+1} = j$.

For example, in Table A4, starting with drought category 1 ($J_n = 1$), the transitions to the next drought category can be determined as follows.

- 56 times that $i = 1, j = 1$
  (e.g., January 1990 to February1990)
- 9 times that $i = 1, j = 2$
  (e.g., August 1991 to September 1991)
- 4 times that $i = 1, j = 3$
  (e.g., May 1996 to June 1996)
- 1 time that $i = 1, j = 4$
  (e.g., May 1990 to June 1990)
- 1 time that $i = 1, j = 5$
  (e.g., November 1998 to December 1998)
- 0 times that $i = 1, j = 6$
- 71 times total that $i = 1$
  (e.g., January 1990, February 1990, etc.)

m_{ij} = number of times that J_n is in state i at time n, and state j at time n+1.

$m_{11} = 56$
$m_{12} = 9$
$m_{13} = 4$
$m_{14} = 1$
$m_{15} = 1$
$m_{16} = 0$

$\Sigma_j m_{ij} = 71.$

Step 5: Determine the transition probabilities by calculating the conditional relative frequencies of the transition counts.

$$\text{transition probability estimates} = \hat{p}_{ij} = \frac{m_{ij}}{\sum_j m_{ij}}$$

$i, j = 1,...,s.$

Using information from Table A5, the transition probabilities become

$\hat{p}_{11} = m_{11} / \Sigma_j m_{1j} = 56/71 = 0.79$
$\hat{p}_{12} = 9/71 = 0.13$
$\hat{p}_{13} = 4/71 = 0.06$
$\hat{p}_{14} = 1/71 = 0.01$
$\hat{p}_{15} = 1/71 = 0.01$
$\hat{p}_{16} = 0/71 = 0.00$
etc.

The full set of transition probabilities are provided in Table A6.

TABLE A4. Drought Category Based on the SPI-3 Indicator for the ACF Basin.

SPI-3 Drought Levels	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
January	1	1	1	1	1	3	1	1	1	3	3
February	1	1	1	1	2	1	1	1	1	4	4
March	1	1	1	1	1	1	1	1	1	3	3
April	1	1	2	1	1	2	1	1	1	6	4
May	1	1	4	2	2	4	1	2	1	4	4
June	4	1	3	5	1	3	3	1	1	2	5
July	4	1	2	5	1	4	4	1	3	1	6
August	6	1	1	6	1	2	3	3	3	2	5
September	5	2	1	5	1	3	1	3	1	5	2
October	4	4	1	1	1	1	1	1	1	3	2
November	3	4	1	1	1	1	1	1	1	2	1
December	3	4	1	1	1	1	2	1	5	2	1

TABLE A5. SPI-3 Transition Counts (m_{ij}) for $J_n = i$ and $J_{n+1} = j$.

Transition Counts	**State "j"**						
State "i"	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
1	56	9	4	1	1	0	71
2	7	2	2	3	2	0	16
3	6	2	3	5	0	1	17
4	1	2	6	4	1	1	15
5	1	1	2	1	1	2	8
6	0	0	0	1	3	0	4

TABLE A6. SPI-3 Transition Probabilities for January 1990 to December 2000.

Transition Probabilities	**State "j"**						
State "i"	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
1	0.79	0.13	0.06	0.01	0.01	0.00	1.00
2	0.44	0.13	0.13	0.19	0.13	0.00	1.00
3	0.35	0.12	0.18	0.29	0.00	0.06	1.00
4	0.07	0.13	0.40	0.27	0.07	0.07	1.00
5	0.13	0.13	0.25	0.13	0.13	0.25	1.00
6	0.00	0.00	0.00	0.25	0.75	0.00	1.00

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